PERGAMON Computers and Mathematics with Applications 39 (2000) 55–56

An International Journal
computers & mathematics with applications

www.elsevier.nl/locate/camwa

Dominono

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Abstract—A two-person game played by alternately marking cells on a square matrix is described. The first player to form a domino *loses*. A symmetry strategy permits second player wins on all even-order fields. Odd-order fields remain unsolved except for the 3×3 on which the second player wins.

Keywords—Two-person games, Dominoes, Symmetry strategy.

Little did Sol Golomb know, when at age 22 he published his first paper on polyominoes (he named the family and its low-order members) that he would open up a vast new field of mathematical recreations. The literature on combinatorial problems, some unusually deep, posed by these fascinating little shapes is now enormous. It is still growing.

In 1979, graph theorist Frank Harary initiated a new area of polyomino problems by generalizing tic-tac-toe. On square fields, two players take turns marking empty cells, each player using his own mark or color. The winner is the first to form a specified polyomino. In what Harary calls "avoidance" games (in contrast to "achievement" games), the first player to form the polyomino loses. My April 1979 "Mathematical Games" column in *Scientific American* [1] was devoted to such games. The column is reprinted, with numerous corrections and additions, in my *Fractal Music, Hypercards, and More* [2]. Sol was such a frequent contributor of fresh material to my columns that it is hard to believe that we never met until I ceased writing it.

If the monomino (single cell) is the objective, the game is a trivial win for the first player on his first move, and a trivial loss if the game is avoidance. If the domino is the objective, the achievement game is an equally trivial win for the first player on his second move, but what about domino avoidance? Surprisingly, a full analysis of this seemingly simple game, which I have named "dominono", turns out to be enormously difficult!

Dominono is played on a square matrix such as the two fields shown in Figure 1. It can be played as a pencil-paper game like tic-tac-toe. One player marks cells with crosses, the other with noughts. They take turns putting their mark on any vacant cell. The person who first forms a domino—that is, who marks two cells that are adjacent horizontally or vertically, but not diagonally—*loses*.

It is more fun to play dominono with checkers or chess pawns, or counters of two different colors. One person uses, say, pennies, the other dimes. They take turns placing one of their coins on a vacant cell. The winner is the first to force his opponent to make a domino.

If the field is 2×2, the second player obviously wins. Indeed, if the field's side has an *even* number of cells, the second player can always win by a symmetry strategy. After each move by his opponent, he simply plays symmetrically opposite on a line from the opponent's move through

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PII: S0898-1221(00)00108-5

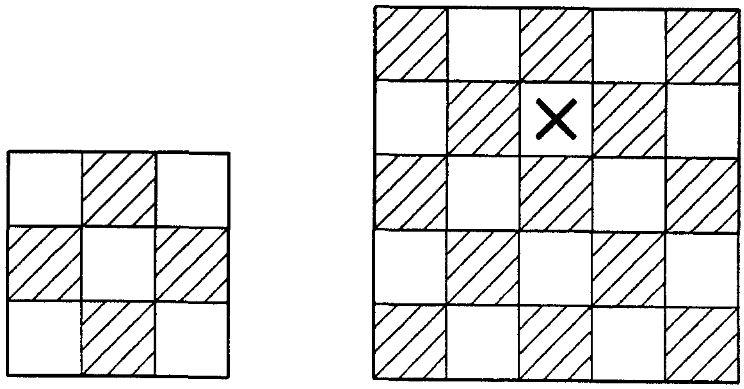

Figure 1.

the field's center. Because of this strategy, only odd-sided fields offer playable games. On the 3×3 field, the second player wins when both sides play their best, but it takes considerable analysis to prove it.

If the opening move is in the center, the second player wins by taking a corner cell, provided it is not diagonally opposite his first move. His third move, on any safe cell, forces the opponent to form a domino on his next move.

If the opening move is on a side cell, the first player clearly cannot later take the center without losing. The second player, therefore, wins by symmetry play.

If the opening move is in a corner, I thought there was no simple winning strategy for the second player until Fred Galvin, a mathematician at the University of Kansas, surprised me by finding a succinct strategy that takes care of *all* opening moves. Here are the rules, based on the coloring shown.

1. Play on a white cell as long as possible.
2. Never let your opponent take two opposite corners.
3. Do not make a domino.

After you run out of white cells, if your opponent has not already lost, he will lose on his next move.

The 5×5 field shown, as well as all larger odd-sided squares, remain unsolved. Perhaps a reader can write a computer program that will decide whether the first or second player can always win, or whether the game is a tie if both sides play "rationally".

It has been conjectured that the second player can always win on all square fields, but this is far from established. Try playing on the 5×5 field. You will quickly see how enjoyable the game is even on so small a field, and also how complex!

Now for a puzzle. Suppose the first player takes the cell marked X on the 5×5 field. Can you prove that this is a losing opening? The simple proof is on page 171.

REFERENCES

1. M. Gardner, Mathematical games, *Scientific American* (April 1979).
2. M. Gardner, *Fractal Music, Hypercards, and More*, W.H. Freeman, (1991).